Systems for constructive reverse mathematics

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In the reverse mathematics over classical logic, most of the results are given on subsystems of Z_2, whose language is for natural numbers and sets of them, besides the works on higher order reverse mathematics such as [8] and reverse recursion theory such as [4].

In the reverse mathematics over intuitionistic logic has not yet had fixed language and systems. Here are some examples:

- **Unformalized reverse mathematics**: They do not use any formal systems and aiming to classify mathematical theorems by the equivalence over Bishop style constructive mathematics [3]. ([2], [5], etc..)
- **Reverse mathematics with function based language**: They use the language for natural numbers and functions over them. Depending on the treatment of function symbols, there are many variants. ([1], [6], [9], [11], etc..)
- **Reverse mathematics with higher order arithmetic**: They use the language for higher order arithmetic. ([7], etc..)
- **Reverse mathematics with first order arithmetic**: They use the language for first order arithmetic. ([10], etc..)

In this talk, we consider the relationship among many systems for constructive reverse mathematics from interpretability and conservativity. We also consider the relationship with systems over classical logic.

References

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